Month	Average E	1910		1911		1912		1913		1914		1915		1916		1917	
		E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C
Average...	91	93	2	89	−2	90	−1	91	0	84	−7	84	−7	94	3	98	7
January...	87	90	3	88	1	87	0	88	1	84	−3	77	−10	86	−1	94	7
February..	88	90	2	86	−2	86	−2	88	0	84	−4	78	−10	87	−1	93	5
March.....	89	92	3	88	−1	88	−1	90	1	84	−5	80	−9	90	1	95	6
April......	91	93	2	89	−2	89	−2	91	0	86	−5	82	−9	92	1	97	6
May......	91	94	3	90	−1	90	−1	92	1	86	−5	83	−8	94	3	98	7
June......	91	94	3	89	−2	90	−1	91	0	85	−6	83	−8	94	3	99	8
July......	90	93	3	88	−2	90	0	91	1	83	−7	83	−7	94	4	98	8
August....	91	94	3	89	−2	91	0	91	0	83	−8	84	−7	96	5	98	7
September	93	96	3	91	−2	94	1	93	0	85	−8	88	−5	98	5	100	7
October...	94	96	2	92	−2	94	0	93	−1	85	−9	89	−5	99	5	102	8
November	93	94	1	91	−2	94	1	91	−2	83	−10	89	−4	99	6	101	8
December..	91	92	1	89	−2	92	1	88	−3	81	−10	89	−2	98	7	99	8

Unemployment in the United States," he has ignored practically all of the data relating to employment fluctuations in occupations other than manufacturing.

Other criticisms are unaffected by Mr. Berridge's second installment. His failure to take account of fluctuations in the available supply of labor, and the fact that his method is so complicated that the resulting index cannot be interpreted in common-sense terms, are serious defects. He is to be congratulated, however, upon his vigorous attack upon a vital problem, and upon having raised important questions of technique in unemployment statistics.

A reply by Dr. Berridge to Dr. Hart's criticism has been received by the Editor, but too late for publication in this issue. It will appear in the December number of the JOURNAL.

PHYSICAL MEASURES AS FUNCTIONS OF FREQUENCY DISTRIBUTIONS

BY M. C. RORTY

A point of view in connection with fundamental physical measurements which is of particular interest to the statistician arises from the very great probability that there are no absolute measures of extension, velocity, and time. Should this be the case, we are concerned, in physical mathematics, with measures which in every case must be derived from the things to be measured. If, for example, we are dealing with velocities, it is obvious that they can be measured only in relation to certain fundamental existing physical velocities. Similarly as to ex-

tension, the primary measure must be found with very great certainty in the dimensions of certain fundamental physical units.

For our present purposes it is not necessary to determine what these velocities and physical units may be. It is sufficient to know that somewhere in the physical scheme of things there must be elements of velocity and extension which are *the* most fundamental. Furthermore, with respect to such fundamental elements of velocity and extension, we can make only three assumptions: first, that all similar elements are of equal magnitude; second, that they have frequency distributions with definite modes; and third, that they have frequency distributions with no modes.

As to the preceding assumptions, it is hardly conceivable that independent elements, either of velocity or of extension, should all be equal in magnitude, in the absence of a previously existing measure by which they might be gaged. And it is almost equally difficult to conceive that they should be arranged in frequency distributions having definite modes, for the existence of such a mode would indicate a preference for some particular magnitude, which preference could hardly exhibit itself in the absence of any established standard of measurement.

If the foregoing reasoning is correct it follows that fundamental physical measures must arise out of frequency distributions having no definite modes. In such distributions the mean value is the natural measure, and the special characteristic of such distributions is that the maximum value is always almost exactly twice the mean.

To give this suggestion concrete form it may be assumed that the velocity of light represents the mean of certain primary velocities in space. On this basis the maximum primary velocity would be twice that of light. This conclusion is entirely concordant with the apparent limitation of material particles to a maximum velocity equal to that of light. In fact, there are reasons, which are, however, not pertinent to the present discussion, why material particles which are made up of a large number of ultimate primary particles should have maximum velocities nearly or exactly equal to the mean velocity of such primary particles.

An interesting feature of this approach to the problem of fundamental physical measurements is that it results in a division of physical elements into two classes. Pure time and space may be infinite, but the velocities and dimensions of material particles must be subject to measures arising out of the frequency distributions of the magnitudes involved. As to the latter, therefore, we are compelled to measure in terms of ratios. In such cases our ordinary notions of infinites become mere mathematical abstractions. The velocities of primary particles

may, in one sense, have a range from zero to infinity; but, if our only measure is one half of infinity, we will, for all practical purposes, assign a value of two to the limiting velocity.

A still more interesting corollary of the hypothesis here presented is that the physicist may know when he has discovered a fundamental physical element by observing whether or not it exhibits a range of magnitudes characterized by a frequency distribution without a mode.

ON THE CORRELATION BETWEEN A VARIABLE AND THE DEVIATION OF AN ASSOCIATED BUT NOT DEPENDENT VARIABLE FROM ITS PROBABLE VALUE

Let r_{xy} be the correlation between two variables, x, y, which are so related to each other physically that y may be considered a dependent variable in the special sense in which this term has been used heretofore. Pearson and I have shown[1] that

$$r_{xz}=\frac{r_{xy}-V_x/V_y}{\sqrt{1-r_{xy}^2+(r_{xy}-V_x/V_y)^2}}$$

where V denotes the coefficient of variation of x and y, and z is to be read as the deviation of y from its probable value, that is

$$z=y-px$$
$$p=\bar{y}/\bar{x}.$$

Now let x and y be associated with a third variable, say w. The correlations r_{wx}, r_{wy}, r_{xy}, r_{xz}, and the partial correlation ${}_wr_{xy}$ may be readily determined. In certain cases the correlation r_{wz} and the partial correlation ${}_wr_{xz}$ may be of service.

$\bar{\bar{z}}=0$; σ_z may be readily determined.[2] We require, therefore, merely the product moment for w and z. This might be determined by working directly with the individual values of the variable $z=y-px$. It is more convenient to proceed as follows:

Let $\Sigma(x)$, $\Sigma(y)$ denote the summation of the values of x and y for the n individuals of any class of w. Let S denote summation throughout the population or summation of the summations within the classes defined by the values of w.

Then within any class of n individuals defined by the value of w

$$\Sigma(z)=\Sigma(y)-p\Sigma(x).$$

For the whole series of $N=S(n)$ individuals

$$S(wz)=S[wS(z)]=S\left\{w[\Sigma(y)-p\Sigma(x)]\right\}.$$

[1] Harris, "The correlation between a variable and the deviation of a dependent variable from its probable value," *Biometrika*, 6 : 438–443. 1909.

[2] Harris, "Further illustrations of the applicability of a coefficient measuring the correlation between a variable and the deviation of a dependent variable from its probable value," *Genetics*, 3 : 328–352. 1918.